Simultaneous Identification and Torque Regulation of Permanent Magnet Synchronous Machines via Adaptive Excitation Decoupling

David M. Reed, Jing Sun and Heath F. Hofmann

Abstract— **Parameter identification and output regulation are generally conflicting objectives. However, in the case of certain overactuated systems, there is an opportunity to achieve these objectives simultaneously. This paper presents a simultaneous identification and adaptive control design methodology for overactuated systems which is applied to the torque regulation problem for permanent magnet synchronous machines. Excitation and control inputs to the system are first designated. The excitation input is then treated as a disturbance which is decoupled from the regulated output via an excitation decoupling control law. Machine parameters are estimated with a normalized gradient-based algorithm, and necessary conditions for parameter convergence are established. Simulation results confirm the necessary conditions for parameter convergence, as well as the effectiveness of the resulting closed-loop torque regulator.**

I. Introduction

Parameter identification and output regulation are typically conflicting objectives. Generally, a trade-off must be made between ensuring that inputs to the system under control are persistently exciting and maintaining tight regulation of "performance" (i.e., regulated) outputs. However, in the case of certain overactuated systems there is an opportunity to achieve these objectives simultaneously. For example, field-oriented output torque regulation in Permanent Magnet Synchronous Machines (PMSMs) constitutes an overactuated control problem in that there are two distinct inputs to the system, the direct-axis voltage input and the quadrature-axis voltage input, and one regulated output, torque. The direct-axis voltage is typically used to set magnetic field (flux) levels in the machine by regulating the direct-axis stator current, while the quadrature-axis voltage is used to regulate the electromagnetic torque by regulating the quadrature-axis stator current.

The PMSM has seen increasing popularity in recent years, particularly in transportation applications as well as industrial applications aimed at induction motor replacement, thanks to its high torque density and high efficiency. However, temperature changes, skin effect, and magnetic saturation lead to changes in the machine parameters which in turn detune the drive system, causing performance degradation. The stator winding resistance is primarily impacted by temperature variations, which can lead to increases in resistance by as much as 100% [1]. While the permanent magnet flux magnitude also varies with temperature, the variation tends to be small, around -0.1% per °C for neodymium ($NdFeB$) magnets [2], which results in a mere 5% variation for a rather large 50°C increase in temperature. Finally, while the electrical frequencies needed to see a significant rise in stator resistance due to skin effect are not typically encountered, high-speed applications using motors with a high pole-pair count may see an impact due to skin effect.

Many different approaches to compensating parameter variations in PMSMs have been proposed by researchers. Steady-state machine models have been used to avoid the additional complexity that comes with using dynamic models for parameter estimation [3], [4]. Open-loop [5], as well as closed-loop [6], [7] approaches have been presented which utilize the method of least-squares for PMSM parameter estimation. The gradient method is used in [8] to provide online estimates of the lumped time-varying disturbances caused by parameter variations. While artificial neural networks have been proposed for online adaptation [9], Lyapunov-based adaptive designs provide an attractive alternative as a stability proof is a byproduct of the design process [10], [11]. However, with the exception of [10], none of these papers proposes a design which specifically considers Simultaneous Identification and Control (SIC) in their design.

The inherent trade-off in SIC designs makes optimization based approaches a natural choice for achieving the SIC objective. In particular, Model-Predictive Control (MPC) is an attractive platform for incorporating the SIC objective because a receding-horizon optimization is inherent to the control method. By incorporating a measure of persistent excitation into the objective function [14], [15], [16], the trade-off between output regulation and excitation levels can be managed by adjusting the relative weighting of these individual "costs." However, while a trade-off is unavoidable in SISO systems, overactuated systems provide an opportunity to circumvent this trade-off by restricting the excitation to the "null-space" of the system. For example, in [17], [18] the authors exploit the overactuated nature of the spacecraft under consideration by restricting the optimized excitation signal to the "null-motion" of craft.

This paper presents a simultaneous identification and control methodology for PMSMs by exploiting the overactuated nature of the machine. An indirect adaptive control design using the certainty equivalence principle is proposed in which a "disturbance decoupling" control law is utilized to prevent the input selected for excitation from perturbing the regulated output. The machine parameters used in this

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excitation decoupling control law are updated via a normalized gradient estimator. Simulation results for a torque regulating controller for PMSMs confirm the effectiveness of the proposed simultaneous identification and control design methodology. Furthermore, while the focus of the paper is on the application of the proposed adaptive excitation decoupling control methodology to PMSM torque regulation, the prospects of generalizing this methodology for overactuated systems are promising.

TABLE I
LIST OF NOTATION AND SPECIAL MATRICES.

Symbol	Description
Electrical Variables	
$v_d^r(t)$	Direct-axis Voltage in Rotor Ref. Frame
$v_q^r(t)$	Quadrature-axis Voltage in Rotor Ref. Frame
$i_d^r(t)$	Direct-axis Current in Rotor Ref. Frame
$i_q^r(t)$	Quadrature-axis Current in Rotor Ref. Frame
R	Stator Winding Resistance
L_d	Direct-axis Stator Self-Inductance
L_q	Quadrature-axis Stator Self-Inductance
Λ_{PM}	Permanent Magnet Flux Linkage
Mechanical Variables	
τ	Three-Phase Electromagnetic Torque
ω_r	Rotor Angular Velocity
$\omega_{re} = \frac{P}{2}\omega_r$	Rotor Electrical Angular Velocity
P	Number of Poles
Special Matrices	
$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$90°$ Rotation Matrix
$e^{-\mathbf{J}\theta}$	Park Transform (Arbitrary Rotation Matrix)

II. FIELD-ORIENTED DYNAMIC MODEL FOR PMSMS

Field-oriented control (FOC) [19] has become the standard approach for high-performance torque regulation in AC machines. Electrical variables, which are normally sinusoidal, are projected into a rotating reference frame using the Park transform [20] to obtain a simplified machine model in which the formerly sinusoidal electrical variables are constant-DC values at steady-state. An additional advantage is that the torque and field generating components of electrical currents are decoupled. The resulting field-oriented machine model is analogous to a separately excited (field-winding) DC machine, where field-and-torque-generating electrical currents are independently controllable.

The control design and parameter estimator presented in this paper are designed using the standard two-phase equivalent model for permanent-magnetic synchronous machines [21]. This model, and the subsequent control design, are derived under the following assumptions:

- **A1.** The machine has a smooth airgap (i.e., slotting effects are not modeled), is balanced in its construction (i.e., the three-phase windings have equal impedance) and has sinusoidally distributed windings;
- **A2.** Core losses are neglected and a linear magnetics model is assumed (i.e., magnetic saturation effects are neglected);
- **A3.** Mechanical dynamics are neglected and so rotor velocity, ω_r, is treated as a known constant;
- **A4.** The only uncertain parameters are resistance, R, and the direct and quadrature inductance, L_d and L_q respectively;
- **A5.** The machine is fed by an "ideal" Voltage Source Inverter (VSI) and switching harmonics are neglected.

The first assumption (**A1**) permits the use of a two-phase equivalent model as it implies that the sum of the three-phase stator currents is zero (i.e., $i_a + i_b + i_c = 0$), and therefore a third state is redundant. The second and third assumptions (**A2**-**A3**) further simplify the model. In particular, since the rotor velocity is measured, and the mechanical dynamics associated with the rotor velocity are slow with respect to the electrical dynamics, we treat ω_{re} as a known constant (**A3**), yielding linear time-invariant machine dynamics. We assume that the permanent magnet flux-linkage, Λ_{PM}, is well known as its variation with temperature tends to be small, and because Λ_{PM} is easily identified offline with the "open-circuit test". For this reason, we assume (**A4**) that R, L_d and L_q are the only uncertain parameters. Finally, by treating the VSI as an ideal "actuator" (**A5**), we neglect voltage dependencies between the *commanded* and *actual* waveforms applied to machine, which may result from switching losses and dead-time effect.

The dynamic model of the PMSM in the rotor reference frame (denoted by the superscript r), in which the direct-axis is aligned with the rotor permanent magnet flux, is given by

$$\frac{d}{dt}\begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L_d} & \omega_{re}\frac{L_q}{L_d} \\ -\omega_{re}\frac{L_d}{L_q} & -\frac{R}{L_q} \end{bmatrix}\begin{bmatrix} i_d^r \\ i_q^r \end{bmatrix}}_{\triangleq f(x)} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L_q} \end{bmatrix}}_{\triangleq g(x)} u_q^r + \underbrace{\begin{bmatrix} \frac{1}{L_d} \\ 0 \end{bmatrix}}_{\triangleq p(x)} u_d^r, \tag{1}$$

with the unmeasured nonlinear torque output mapping,

$$\tau = \underbrace{\frac{3P}{4}\left[\left(L_d - L_q\right) i_d^r + \Lambda_{PM}\right] i_q^r}_{\triangleq h(x)}, \tag{2}$$

where the direct and quadrature currents are the states (i.e., $x = [i_d^r \; i_q^r]^T$) of the system and the direct and quadrature voltage inputs to the system are assumed to include a EMF cancellation term, i.e., $v_q^r = u_q^r + \Lambda_{PM}\omega_{re}$, and $v_d^r = u_d^r$ for consistency. In conventional terms, these equations (1)-(2) represent an LTI system with nonlinear regulated (or "performance") output mapping.

III. EXCITATION DECOUPLING VIA STATE-FEEDBACK

The SIC approach presented in this paper is based upon a certainty equivalence design in which an "excitation input decoupling" control law is used to prevent the excitation

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input signal from perturbing the regulated output. This excitation decoupling control law is derived by reformulating the problem as a "disturbance decoupling" problem [22].

A. Statement of the Control Objective

The control inputs to the PMSM are the direct and quadrature-axis voltages, u_d^r and u_q^r, and the (unmeasured) regulated output is electromagnetic torque, τ. Thus, the PMSM constitutes an overactuated[1] system. Our control objective is to simultaneously achieve parameter identification and asymptotic output regulation in an overactuated system, namely the PMSM. This is accomplished by using an adaptive excitation (disturbance) decoupling control design in which one input of the overactuated system is designated as the "excitation input" used to ensure that the PMSM dynamics are persistently excited for parameter convergence, and the other input is used for torque output regulation. The excitation decoupling control law ensures that the perturbations in the regulated output go to zero asymptotically as the machine parameters converge to their true values. Once identified, the presence of the excitation signal ensures that the parameter estimator will track any changes in the parameters.

B. Review of Disturbance Decoupling

To apply this solution to the simultaneous identification and control problem, we treat the excitation input as a "measured" disturbance, and derive a state-feedback controller which decouples the excitation input from the regulated output, provided that the system parameters are well known. For convenience, we will first review the general solution for a class of nonlinear systems [22] before applying the result to the PMSM torque regulation problem.

Consider a nonlinear system of the form

$$\Sigma : \begin{cases} \dot{x} = f(x) + g(x)u + p(x)w, \\ y = h(x), \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}$ is the "regulated" (or "performance") output, $u \in \mathbb{R}$ is the input and $w \in \mathbb{R}$ is the "disturbance" input which is to be decoupled.

Given measurements of the full state vector, x, as well as the disturbance, w, it is possible to decouple the disturbance from the output, y, using a state-feedback law of the form $u = \alpha(x) + \beta(x)v + \gamma(x)w$, where v is a control reference input which will be designed to yield stable linear closed-loop dynamics, provided that,

$$L_p L_f^i h(x) = 0 \quad \text{for all} \quad 0 \le i \le \rho - 2, \tag{4}$$

$$L_p L_f^{\rho-1} h(x) = -L_g L_f^{\rho-1} h(x)\gamma(x), \tag{5}$$

for all x in the neighborhood of the equilibrium, x^o, where $L_f^i h(x)$ denotes the i^{th} Lie derivative of $h(x)$ projected along $f(x)$ and ρ is the "relative degree" of the system, Σ. Note that the second condition is easily satisfied by solving for $\gamma(x)$ and including the term in the feedback law. If these conditions are satisfied for a given plant, a feedback law which achieves disturbance decoupling, is given by:

$$u = -\frac{L_f^\rho h(x)}{L_g L_f^{\rho-1} h(x)} - \frac{L_p L_f^{\rho-1} h(x)}{L_g L_f^{\rho-1} h(x)} w + \frac{v}{L_g L_f^{\rho-1} h(x)}. \tag{6}$$

The design of v is best understood by considering the system in the "normal form" [22], which is obtained by defining new coordinates such that,

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{\rho-1} &= z_\rho \\ \dot{z}_\rho &= b(\xi,\eta) + a(\xi,\eta)u + d(\xi,\eta)w \\ \dot{\eta} &= q(\xi,\eta) + k(\xi,\eta)w \\ y &= z_1 \end{aligned}$$

where $\xi = [z_1 \cdots z_\rho]^T$ and $\eta = [z_{\rho+1} \cdots z_n]^T$. Note that the term $\dot{\eta} = q(0,\eta)$ represents the "zero dynamics" of the system, which must be stable (i.e., minimum phase) to ensure that the closed-loop design is internally stable. In the new coordinate system (i.e., normal form) the state-feedback law (6) takes the form,

$$u = -\frac{b(\xi,\eta)}{a(\xi,\eta)} - \frac{d(\xi,\eta)}{a(\xi,\eta)} w + \frac{v}{a(\xi,\eta)},$$

and the closed-loop system takes on the structure depicted in Fig. 1, isolating the output from the disturbance input and the system states which are influenced by the disturbance input.

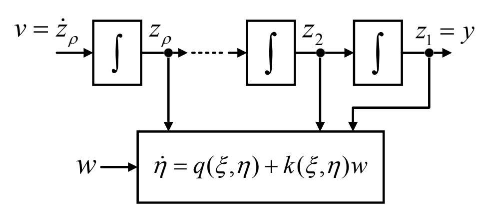

Fig. 1. Block diagram of closed-loop system after disturbance decoupling (adapted from [22]).

With Fig. 1 in mind, our choice of the control, v, is rather intuitive,

$$\begin{aligned} v &= -(c_0 z_1 + \cdots + c_{\rho-1} z_\rho) + \tilde{y} \\ &= -\left(c_0 h(x) + \cdots + c_{\rho-1} L_f^{\rho-1} h(x)\right) + \tilde{y}, \end{aligned} \tag{7}$$

where $\tilde{y}$ is the reference input and the coefficients $c_0, \cdots, c_{\rho-1}$ are selected to yield the desired linear dynamics with characteristic equation,

$$s^\rho + c_{\rho-1} s^{\rho-1} + \cdots + c_1 s + c_0 = 0. \tag{8}$$

It should be noted that in general, the solution to the disturbance decoupling problem presented here, and its asymptotic stability properties, are *local* results (i.e., they hold only for a neighborhood of the equilibrium point, x^o).

[1]Defined here as a system in which the number of control inputs is strictly greater than the number of regulated outputs.

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C. Excitation Decoupling for PMSMs

To apply the disturbance decoupling solution to our problem, we designate the quadrature-axis voltage, u_q^r, as the control input, as it has the most control authority over the torque output, and the direct-axis voltage, u_d^r, as the "excitation" (or "disturbance") input to be decoupled. We therefore treat the PMSM as a SISO system with measured disturbance input, u_d^r.

The relative degree of the PMSM system (1)-(2) from either input (u_d^r or u_q^r) to the output, τ, is one (i.e., $\rho = 1$ for the PMSM), which is easily verified by differentiating the output (2) with respect to time. Therefore, we need only include the following feed-forward term

$$\gamma(x) = \frac{L_q}{L_d} \frac{(L_d - L_q) i_q^r}{(L_d - L_q) i_d^r + \Lambda_{PM}}$$

in the excitation decoupling control law to ensure that the conditions (4)-(5) are satisfied. Furthermore, since the PMSM dynamics (1)-(2) are minimum phase, we may apply the disturbance decoupling results from the previous section.

While our focus is on the design of the control input, u_q^r, in practice it is beneficial to cancel the cross coupling term in the direct-axis dynamics as it will lead to resonant behavior, as well as a large steady-state direct-axis current, at high rotor velocities. Therefore, we include the following feedback term for the direct-axis,

$$u_d^r = -\omega_{re} \hat{L}_q i_q^r + \hat{R} u_e, \tag{9}$$

where u_e is the "excitation input" which is scaled by the estimated resistance so that it corresponds to the magnitude of the steady-state direct-axis current generated by the direct-axis command voltage. Computing the necessary Lie derivatives specified in (6), we obtain the excitation decoupling control law for PMSMs:

$$u_q^r = \hat{R} i_q^r + \omega_{re} \hat{L}_d i_d^r - \frac{\hat{R}}{\hat{L}_d} \frac{\hat{L}_q \hat{\Delta}_L i_q^r}{\hat{\Delta}_L i_d^r + \Lambda_{PM}} (u_e^r - i_d^r) + \frac{4 \hat{L}_q}{3P \left(\hat{\Delta}_L i_d^r + \Lambda_{PM} \right)} v, \tag{10}$$

where the "hat" ($\hat{\cdot}$) designates parameters which will be adaptively estimated and $\hat{\Delta}_L = \hat{L}_d - \hat{L}_q$ (for compactness).

Finally, we design the control input, v, in (10) as follows:

$$v = -\lambda h(x) + \lambda \tilde{\tau}, \tag{11}$$

which yields the following first-order (input-output) closed-loop dynamics,

$$\Sigma_{cl} : \begin{cases} \dot{z} = -\lambda z + \lambda \tilde{\tau} \\ y = z = h(x) \end{cases} \tag{12}$$

where $\lambda > 0$ is a control gain which sets the closed-loop bandwidth. Thus, the disturbance and its associated dynamics are rendered unobservable in the output, z. Finally, we compute the value for λ based on our desired rise-time of 2 milliseconds and the following relationship for first-order systems [23],

$$\lambda = \frac{1.8}{t_r},$$

where t_r is our desired rise time (i.e., $t_r = 2$ msec).

Remark: A comprehensive stability analysis will not be pursued in this paper. However, once parameter convergence is obtained, which is guaranteed by the algorithm proposed in the next section, stability of the closed-loop system will follow from the non-adaptive disturbance decoupling control. Therefore, for the adaptive control problem, stability can be assured if there is no finite escape time.

IV. Parameter Identification

Estimates of the machine parameters used in the excitation decoupling control law (10) are provided by a normalized gradient-based algorithm. The resistance, as well as the direct and quadrature inductances, are directly estimated by the algorithm. A block diagram of the overall adaptive control system is provided in Fig. 2.

A. Parameteriziation and Gradient-based Estimator

To formulate the parameter estimator, we first construct a linear parameterization for the system model (1),

$$\vec{z} = \mathbf{\Phi}^T \vec{\theta}, \tag{13}$$

where $\vec{z} = [u_d^r \ \ u_q^r]^T$ is the "measurement" vector, $\vec{\theta} = [R \ \ L_d \ \ L_q]^T$ is the parameter vector, and the regressor matrix is given by

$$\mathbf{\Phi}^T = \begin{bmatrix} \vec{\phi}_d^T \\ \vec{\phi}_q^T \end{bmatrix} = \begin{bmatrix} i_d^r & \frac{d}{dt} i_d^r & -\omega_{re} i_q^r \\ i_q^r & \omega_{re} i_d^r & \frac{d}{dt} i_q^r \end{bmatrix}.$$

In order to avoid direct computation of derivatives in the regressor matrix, we filter each side of (13) by a stable first-order filter [24], i.e.,

$$\{M(s)\} \, \vec{z} = \{M(s)\} \, \mathbf{\Phi}^T \vec{\theta},$$

where

$$\{M(s)\} = \left\{ \frac{K_f}{s + K_f} \right\}$$

is the transfer function representation of a stable first-order filter (i.e., $K_f > 0$) which operates on the individual elements of $\vec{z}$ and $\mathbf{\Phi}$.

The parameter estimates are obtained by integrating the following expression,

$$\begin{aligned} \dot{\hat{\vec{\theta}}} &= \mathbf{\Gamma} \mathbf{\Phi} \vec{e}_z \\ &= \mathbf{\Gamma} \begin{bmatrix} \vec{\phi}_d & \vec{\phi}_q \end{bmatrix} \begin{bmatrix} e_{zd} \\ e_{zq} \end{bmatrix} \\ &= \mathbf{\Gamma} \left(\vec{\phi}_d e_{zd} + \vec{\phi}_q e_{zq} \right) \end{aligned} \tag{14}$$

where $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0$ is the adaptation gain matrix and $\vec{e}_z = [e_{zd} \ \ e_{zq}]^T$ is the normalized estimator error vector, whose entries are given by,

$$e_{zd} = \frac{u_d^r - \vec{\phi}_d^T \hat{\vec{\theta}}}{1 + \vec{\phi}_d^T \vec{\phi}_d}, \tag{15}$$

$$e_{zq} = \frac{u_q^r - \vec{\phi}_q^T \hat{\vec{\theta}}}{1 + \vec{\phi}_q^T \vec{\phi}_q}. \tag{16}$$

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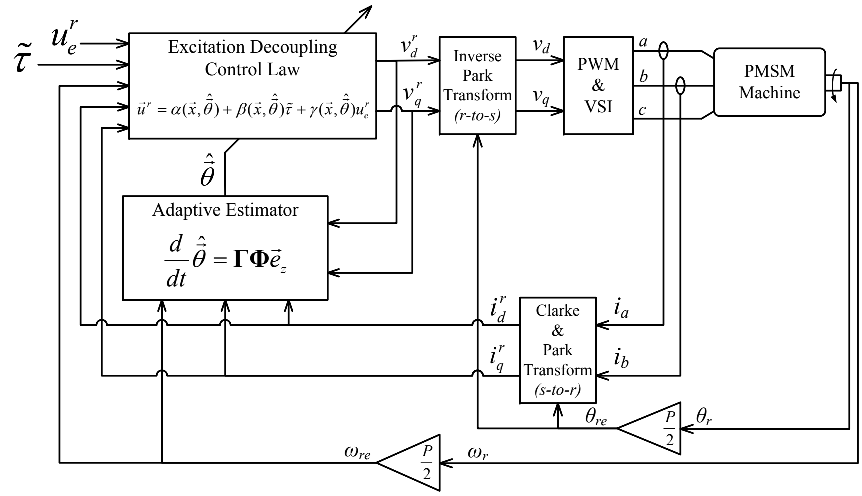

Fig. 2. Block diagram of the closed-loop system with proposed adaptive excitation decoupling controller.

Finally, we note that the rows of the regressor matrix, $\mathbf{\Phi}$, are scaled to ensure that the regressor is well-conditioned. This is important as the large differences in the order-of-magnitude between resistances and inductances will lead to a poorly-conditioned regressor if left unscaled.

B. Sufficient Richness Analysis

To establish requirements on the excitation and control inputs, an analysis of the linearized closed-loop system is performed. Specifically, we will show that the control input alone is insufficient for parameter identification, and that a nonzero reference input (i.e., torque command) is required as well.

Assuming that the parameters are well known[2], the linearized (via Taylor series expansion) closed-loop dynamics for the PMSM (1) with feedback (10) are given by,

$$\frac{d}{dt}\begin{bmatrix}\delta i_d^r\\ \delta i_q^r\end{bmatrix} = \begin{bmatrix}-\frac{R}{L_d} & 0\\ A_{21}(X,U) & -\lambda\end{bmatrix}\begin{bmatrix}\delta i_d^r\\ \delta i_q^r\end{bmatrix} + \begin{bmatrix}0\\ \frac{4\lambda}{3P\left(\Delta_L I_d^r + \Lambda_{PM}\right)}\end{bmatrix}\delta\tilde{\tau} + \begin{bmatrix}\frac{R}{L_d}\\ -\frac{R}{L_d}\frac{\Delta_L I_q^r}{\Delta_L I_d^r + \Lambda_{PM}}\end{bmatrix}\delta u_e^r, \tag{17}$$

where I_d^r and I_q^r are the steady-state (or equilibrium) values of the direct and quadrature current respectively, U_e^r and $\tilde{T}$ are the steady-state (or equilibrium) set-points for the excitation and torque reference inputs respectively, and the term,

$$A_{21}(X,U) = \frac{R}{L_d}\frac{\Delta_L}{\Delta_L I_d^r + \Lambda_{PM}}\left(I_q^r\Lambda_{PM} + \Delta_L I_q^r U_e^r - \lambda\frac{L_d}{R}\frac{4}{3P}\tilde{T}\right),$$

which is a function of the equilibrium states (I_d^r, I_q^r) and inputs $(U_e^r, \tilde{T})$, is defined for compactness. Note that the lower-case delta δ denotes "small" perturbations from the equilibrium or set-point values. Inspection of (17) reveals that the closed-loop eigenvalues are $-R/L_d$ and $-\lambda$, as expected. However, the mode associated with $-R/L_d$ has been rendered unobservable with respect to the torque output, and so the eigenvalue at $-\lambda$ determines the closed-loop bandwidth.

While the complexity of $A_{21}(X,U)$ prevents us from performing a clear analysis of how the excitation input influences the conditioning of the regressor matrix, we are able to compute a transfer function relationship between the regressor elements and the reference input, $\delta\tilde{\tau}$. First, we derive the transfer function matrix relating the input and the states,

$$\begin{aligned}\delta\vec{i}^r(s) &= (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\delta\tilde{\tau}(s)\\ &= \frac{4}{3P\left(\Delta_L I_d^r + \Lambda_{PM}\right)}\begin{bmatrix}0\\ \frac{\lambda}{s+\lambda}\end{bmatrix}\delta\tilde{\tau}(s)\\ &= c_\tau\begin{bmatrix}0\\ \frac{\lambda}{s+\lambda}\end{bmatrix}\delta\tilde{\tau}(s),\end{aligned}$$

where $c_\tau = 4/3P\left(\Delta_L I_d^r + \Lambda_{PM}\right)$ is a constant scalar. Next, we derive the transfer function relationship between the elements of the regressor matrix and the reference input,

$$\mathbf{\Phi} = \mathbf{H}(s)\delta\tilde{\tau}(s) = c_\tau\begin{bmatrix}0 & \frac{\lambda}{s+\lambda}\\ 0 & 0\\ -\frac{\omega_{re}\lambda}{s+\lambda} & \frac{s\lambda}{s+\lambda}\end{bmatrix}\delta\tilde{\tau}(s). \tag{18}$$

To analyze whether or not the regressor matrix is persistently excited under a particular input, we will use the following theorem, adapted from [24].

Theorem: *Consider the equation $\mathbf{\Phi} = \mathbf{H}(s)u$ where $\mathbf{H}(s)$ is a proper transfer matrix with stable poles and $\mathbf{\Phi} \in \mathbb{R}^{n\times m}$ with $m < n$. Let $u : \mathbb{R}^+ \mapsto \mathbb{R}$ be stationary and assume*

[2]Intuitively, we expect this to be the worst-case-scenario since any error in the estimated parameters is expected to provide additional "information" for identification.

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that $H(j\omega_1), ..., H(j\omega_n)$ are linearly independent columns of $\mathbf{H}(s)$ on $\mathcal{C}^n$ for all $\omega_1, \omega_2, ..., \omega_n \in \mathbb{R}$, where $\omega_i \neq \omega_k$ for $i \neq k$. Then $\mathbf{\Phi}$ is persistently exciting if, and only if, u is sufficiently rich of order n (i.e., u contains at least $\frac{n}{2}$ distinct frequencies).

Inspection of (18) reveals that regardless of the choice of reference input, $\delta\tilde{\tau}(s)$, there does not exist a set of columns of $\mathbf{H}(j\omega)$ for which $H(j\omega_1), ..., H(j\omega_n)$ are linearly independent for any ω_i, with $\omega_i \neq \omega_k$. Therefore, because of the row of zeros in (18) we conclude that the control input alone is insufficient for parameter identification [24]. It should be noted that the presence of the filter, $M(s)$ does not affect the conclusions of this analysis[3], and is therefore neglected for simplicity. Intuitively, we expect that the direct-axis inductance, L_d, which is associated with the second row of the regressor matrix, will fail to converge without additional excitation provided by the excitation input.

In addition to the need for the excitation input, a requirement on the reference input set-point, $\tilde{T}$, is established as well. If we consider the torque expression (2), we see that a zero torque command implies that the quadrature current is equal to zero at steady-state. Under this steady-state operating condition, we have $A_{21}(X, U) = 0$, considerably simplifying our analysis. We compute the transfer function matrix relating the excitation input to the states, assuming that $\tilde{T} = I_q^r = 0$,

$$\begin{aligned}\delta\vec{i}^r(s) &= (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\delta u_e^r(s)\\ &= \frac{R}{L_d}\begin{bmatrix}\frac{1}{s+\frac{R}{L_d}}\\ 0\end{bmatrix}\delta u_e^r(s),\end{aligned}$$

and use this relationship to relate the excitation input to the elements of the regressor matrix,

$$\mathbf{\Phi} = \mathbf{H}(s)\delta u_e^r(s) = \frac{R}{L_d}\begin{bmatrix}\frac{1}{s+\frac{R}{L_d}} & 0\\ \frac{s}{s+\frac{R}{L_d}} & \frac{\omega_{re}}{s+\frac{R}{L_d}}\\ 0 & 0\end{bmatrix}\delta u_e^r(s). \quad (19)$$

Similar to the preceding analysis, inspection of (19) reveals that regardless of the choice of excitation input, $\delta u_e^r(s)$, there does not exist a set of columns of $\mathbf{H}(j\omega)$ for which $H(j\omega_1), ..., H(j\omega_n)$ are linearly independent for any ω_i, with $\omega_i \neq \omega_k$, and therefore full parameter convergence cannot be obtained when the reference input set-point, $\tilde{T}$, is equal to zero. In this case we expect that the quadrature-axis inductance, L_q, which is associated with the bottom row of the regressor matrix, will fail to converge.

To summarize, we have used the analysis of the linearized closed-loop system presented in this section to establish the following two conditions related to parameter convergence:

1) Perturbations in the reference input, $\delta\tilde{\tau}$, alone cannot provide sufficient excitation for full parameter convergence (i.e., we need to utilize the excitation input as well);
2) A necessary condition for full parameter convergence is that the reference input set-point $\tilde{T} \neq 0$.

Taken together, these conditions suggest that *if* the machine parameters are identifiable, then a sufficiently rich excitation input combined with a nonzero torque command will achieve full parameter convergence.

V. Simulation Results

We have validated the proposed adaptive excitation decoupling control methodology for SIC in Matlab/Simulink simulations using a dynamic model of the PMSM. Parameters provided in Table II were used in all simulations except where noted otherwise.

TABLE II
SIMULATION PARAMETERS.

Description	Value
Electrical Machine Parameters:	
R	102.8 $m\Omega$
L_d	212.3 μH
L_d	424.6 μH
Λ_{PM}	12.644 mV-s
P	10
Control Design Parameters:	
λ	900
$\mathbf{\Gamma}$	diag([16 80 40])
K_f	1000
u_e^r	$1.5\,(\sin(\omega_{pe}t) + \sin(0.5 \cdot \omega_{pe}t))$
ω_{pe}	363 rad/sec
Simulation Settings:	
Solver	ode4 (Fixed-step Runge-Kutta)
Step Size	10 μ-sec

A. Conditions for Parameter Convergence

By exploiting the overactuated nature of the PMSM, we are able to ensure that the machine dynamics are persistently excited, regardless of the harmonic content (i.e., “sufficient richness”) of the reference input. Our only requirement on the reference input for full parameter convergence is that the torque command be nonzero, as the analysis in the preceding section indicated. To illustrate the necessary conditions which have been derived, Simulink simulations are run which examine parameter convergence when the excitation input is set to zero, and in which the torque (control) command is set to zero.

In Fig. 3, we see that without the additional information provided by the excitation input, $u_e = 0$, the estimate of the direct-axis inductance, $\hat{L}_d$, settles to an incorrect value, as our analysis predicted. This scenario serves as our baseline adaptive control design in which the overactuation in the system *is not* exploited. This essentially gives us the feedback linearization portion of the controller and a fair basis with which to compare. Despite the rich harmonic content of

[3]Given a persistently exciting u with $\dot{u}$ bounded, and a stable, minimum phase, proper transfer function $M(s)$, it follows that $y = M(s)u$ is also persistently exciting [24].

the torque command, the estimated parameters do not fully converge to their true values.

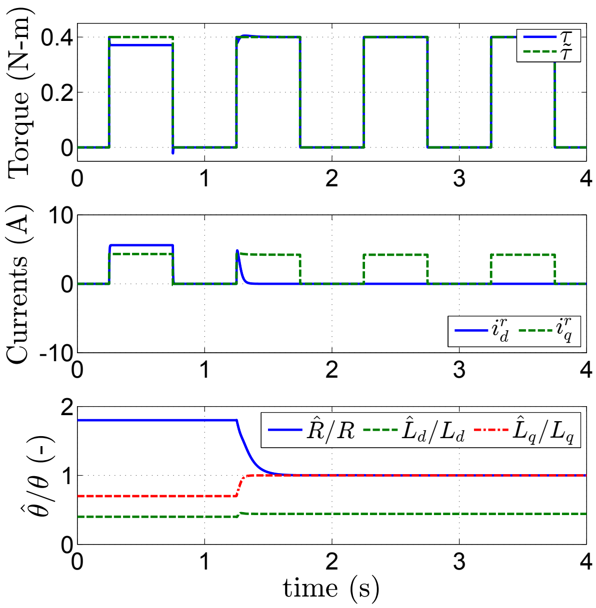

Fig. 3. Simulation of closed-loop adaptive system without persistently exciting input, leading to incomplete convergence (adaptation turned "on" at t = 1 sec).

However, in Fig. 4 we see that when the torque command is set to zero the estimated machine parameters again fail to converge fully to their true values, despite the presence of a persistently exciting excitation input, $u_e = 1.5\,(\sin(\omega_{pe}t) + \sin(0.5 \cdot \omega_{pe}t))$. As our analysis predicted, the zero torque command results in a lack of sufficient richness for the quadrature-axis inductance estimate, $\hat{L}_q$, to converge to its true value.

B. Closed-loop Performance

The main objective in this paper is to demonstrate an adaptive control methodology for overactuated systems which is capable of achieving simultaneous identification of parameters and control of a regulated output. This is achieved by exploiting the overactuated nature of a PMSM by designating one input as an excitation input, which is designed to ensure that the system is persistently excited for parameter identification, and the other input as the control input used for torque regulation.

In Fig. 3, we see that without leveraging the extra degree of freedom resulting from overactuation (i.e., utilizing the excitation input) the estimated parameters converge to a set where, despite the fact that the control error goes to zero, the estimate of L_d stagnates at an incorrect value. This is the typical scenario for adaptive control in which the adaptation drives the control error to zero, but due to a lack of persistent excitation, the parameters fail to completely converge to their true values.

However, when we use the overactuated nature of the PMSM to our advantage, we are able to ensure that the system is persistently excited and so all of the parameters

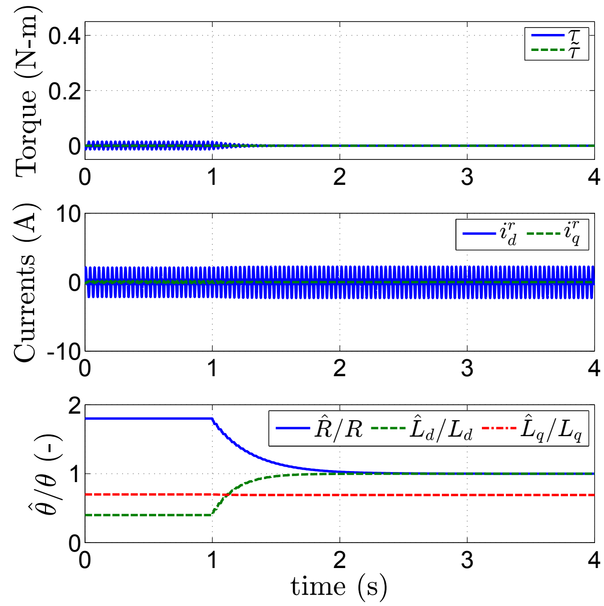

Fig. 4. Simulation of closed-loop adaptive system with zero torque command input, leading to incomplete convergence (adaptation turned "on" at t = 1 sec).

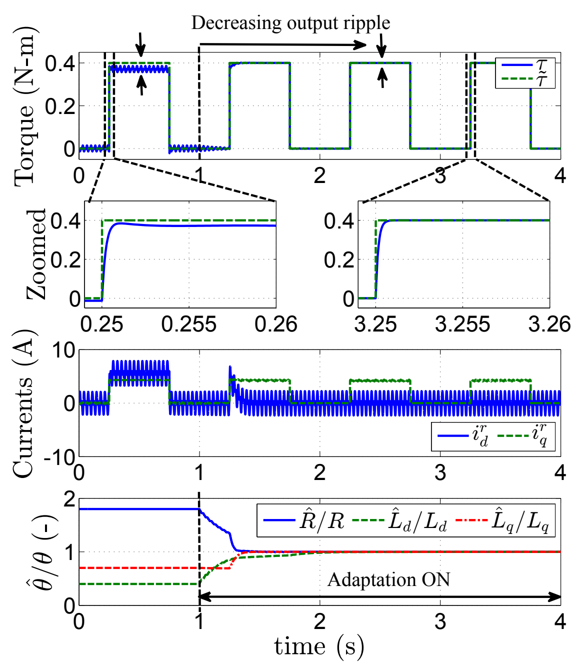

Fig. 5. Simulation of closed-loop adaptive system at a fixed rotor velocity of 2000 rpm with excitation input (adaptation turned "on" at t = 1 sec).

converge to their true values. Inspection of the results in Fig. 5 confirm that the closed-loop system performs very well, with torque perturbations due to the excitation input vanishing as the estimated parameters converge to their true values.

Finally, to demonstrate that the closed-loop adaptive excitation decoupling controller does exhibit robustness to uncertainty as well, simulations are run which include zero-

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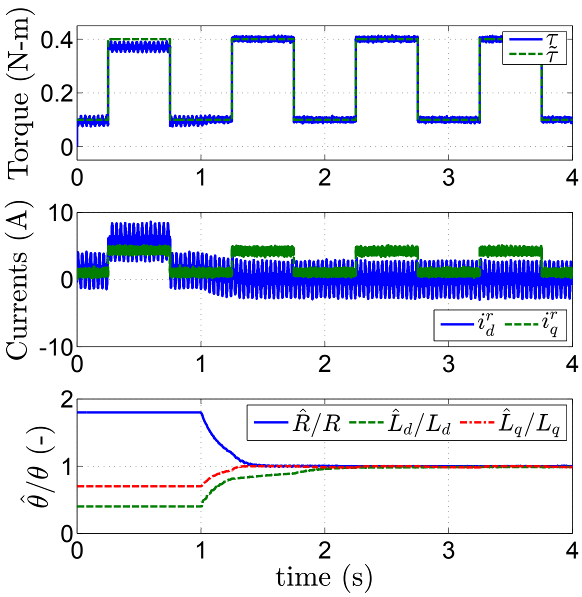

Fig. 6. Simulation of closed-loop adaptive system at a fixed rotor velocity of 2000 rpm with zero-mean Gaussian noise added to the current measurements.

mean Gaussian noise on the stator current measurements. Inspection of these results, presented in Fig. 6, indicate that despite the presence of measurement noise, the parameters converge to their true values. It should be noted that in practice limiting the bandwidth of the bandwidth of the adaptive estimator will improve performance in presence of measurement noise. Additionally, while the simulations presented here do not include any robustness modification to the adaptive update law (14), in an experimental implementation, the addition of a robustness modification such as a switching-sigma or projection is advised [24].

VI. Conclusion

This paper presented a simultaneous identification and control methodology for PMSMs which exploits the over-actuated nature of the machine. An indirect adaptive control design using the certainty equivalence principle was developed in which a "disturbance decoupling" control law is utilized. The machine parameters used in this excitation decoupling control law are updated via a normalized gradient estimator, and analysis of the linearized closed-loop system established necessary conditions for full parameter convergence. Simulation results confirm the effectiveness of the proposed SIC design methodology. Finally, while the focus of the paper is on the application of the proposed adaptive excitation decoupling control methodology to PMSM torque regulation, the prospects of generalizing this methodology for overactuated systems are promising.

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