A Receding Horizon Approach to Simultaneous Identification and Torque Control of Permanent Magnet Synchronous Machines

David M. Reed, Jing Sun and Heath F. Hofmann

Abstract— **Output regulation and the identification of plant parameters are generally conflicting objectives. However, over-actuated systems provide an opportunity to achieve identification and control objectives simultaneously, with minimal compromise. This paper presents an optimization-based simultaneous identification and control (SIC) methodology for permanent magnet synchronous machines (PMSMs) which exploits the over-actuated nature of the machine. A receding horizon control allocation (RHCA) approach is used which includes a metric for maximizing the excitation characteristics of the generated reference current trajectories. The reference currents produced by the RHCA are fed to a lower-level adaptive current regulator which ensures asymptotic tracking of a reference model. The importance of reformulating the RHCA problem to include past input and state data, in addition to predicting future input and state trajectories, is discussed. Simulation results demonstrating the effectiveness of the proposed RHCA-SIC methodology, as well as the effects of neglecting past input and state data, are presented.**

I. Introduction

Output regulation and the identification of plant parameters are generally conflicting objectives, necessitating a trade-off between ensuring that control inputs are persistently exciting for parameter identification, and minimizing the regulated output error. This trade-off between identification and control makes optimization-based design methodologies a natural choice for Simultaneous Identification and Control (SIC). Model Predictive Control (MPC) [1], which has seen a rapid growth in popularity in recent years, provides a natural platform for SIC due in part to its inherent optimization and constraint handling. In the MPC framework, a metric for excitation is incorporated into the optimization to encourage the generation of persistently exciting control signals [2]–[7]. The trade-off between identification and control may then be managed by tuning the weighting (or penalties) placed on excitation and regulation metrics.

While a trade-off must be made between identification and control in designs for SISO systems, over-actuated systems provide an opportunity to achieve identification and control objectives simultaneously, with minimal (if any) compromise. For example, the authors of [8], [9] exploit over-actuation in a spacecraft with a redundant reaction wheel, restricting excitation to the "null-motion" of craft. Similarly, in [10] the authors take advantage of over-actuation in an electric vehicle with in-wheel motors to improve the identification of the tire-road coefficient of friction. In this paper, we consider torque regulation for permanent magnet synchronous machines (PMSMs), which constitutes an over-actuated problem as there are multiple inputs (i.e., direct and quadrature-axis voltages) and a single regulated output (i.e., torque).

Interest in electric and hybrid vehicles, as well as induction machine replacement in industrial settings, has driven a rise in the popularity of PMSMs, due to their high efficiency and torque density. While easier to control than induction machines, PMSM controllers still suffer performance degradation due to variations in machine parameters, particularly in high-performance applications. Temperature changes, skin effect, and magnetic saturation all contribute to variations in the machine parameters. The stator resistance may increase by as much as 100% [11], primarily due to temperature variations, but also skin effect when high electrical frequencies are involved[1]. While variations in the stator resistance can be compensated without the use of parameter estimation, variations in the magnetic parameters (i.e., inductances and permanent magnet flux linkage) will have a direct impact on torque production. Since the torque is not measured, these variations can only be compensated by updating the parameter values in the torque control algorithm. A variety of strategies for compensating parameter variations in PMSMs have been proposed in the literature [12]–[21]. However, only [20], [21] specifically consider the SIC problem and over-actuation in the proposed control designs.

Previous work [20], [21] explored more traditional control designs for achieving SIC in PMSMs with emphasis on exploiting over-actuation. In this paper, we present an optimization-based simultaneous identification and control methodology for permanent magnet synchronous machines which exploits the over-actuated nature of the machine. A receding horizon control allocation (RHCA) is used which includes a metric for maximizing the excitation characteristics of the generated reference current trajectories. The reference currents produced by the RHCA are fed to a lower-level adaptive current regulator which ensures asymptotic tracking of a reference model. After reviewing the PMSM dynamics and our control objectives, we discuss the proposed control architecture and introduce the (static) control allocation problem for PMSM torque regulation. Metrics for optimizing the conditioning of the Fisher information matrix and

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This work was sponsored by the U.S. Office of Naval Research (ONR) under Grant No. 00014-11-1-0831.

[1]Typically encountered in high speed applications and when using machines with a high number of poles.

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

their application to generating persistently exciting inputs are then discussed, as well as the necessary modifications to the control allocation problem, needed for excitation maximization, which lead to the RHCA formulation. Finally, the crucial role of past input and state data in the RHCA-SIC algorithm is discussed, and simulation results demonstrating the effectiveness of the methodology, as well as the need for past data, are presented.

TABLE I

LIST OF COMMON NOTATION.

Symbol	Description
Electrical Variables	
$v_d^r(t)$	Direct-axis Voltage in Rotor Ref. Frame
$v_q^r(t)$	Quadrature-axis Voltage in Rotor Ref. Frame
$i_d^r(t)$	Direct-axis Current in Rotor Ref. Frame
$i_q^r(t)$	Quadrature-axis Current in Rotor Ref. Frame
R	Stator Winding Resistance
L_d	Direct-axis Stator Self-Inductance
L_q	Quadrature-axis Stator Self-Inductance
Λ_{PM}	Permanent Magnet Flux Linkage
Mechanical Variables	
τ	Three-Phase Electromagnetic Torque
ω_r	Rotor Angular Velocity
$\omega_{re} = \frac{P}{2}\omega_r$	Rotor Electrical Angular Velocity
P	Number of Poles
Special Matrices	
$\mathbf{I}$	Identity Matrix
$\mathbf{0}$	Zero Matrix

II. Plant Dynamics and Control Objectives

A. Two-phase Equivalent Model of PMSM

The standard two-phase equivalent model for permanent-magnet synchronous machines [22] is given by

$$\Sigma : \begin{cases} \dot{\vec{i}}^r = \mathbf{A}(t,\vec{\theta})\vec{i}^r + \mathbf{B}(\vec{\theta})\vec{v}^r - \vec{d}(t,\vec{\theta}), \\ \tau = h(\vec{i}^r,\vec{\theta}), \end{cases} \tag{1}$$

with,

$$\mathbf{A}(t,\vec{\theta}) = \begin{bmatrix} -\frac{R}{L_d} & \omega_{re}(t)\frac{L_q}{L_d} \\ -\omega_{re}(t)\frac{L_d}{L_q} & -\frac{R}{L_q} \end{bmatrix}, \mathbf{B}(\vec{\theta}) = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix},$$

$$\vec{d}(t,\vec{\theta}) = \begin{bmatrix} 0 \\ \omega_{re}(t)\Lambda_{PM} \end{bmatrix},$$

where $\vec{i}^r = [i_d^r \; i_q^r]^T$ is the state vector, $\vec{v}^r = [v_d^r \; v_q^r]^T$ is the input vector, $\vec{\theta} = [R \; L_d \; L_q \; \Lambda_{PM}]^T$ is the parameter vector, and the *unmeasured* nonlinear torque output mapping is given by

$$h(\vec{i}^r,\vec{\theta}) = \frac{3P}{4}\left[(L_d - L_q)\, i_d^r + \Lambda_{PM}\right] i_q^r. \tag{2}$$

Additionally, we note that the superscript "r" identifies signals which are expressed in the rotor reference-frame, and in which the direct-axis is aligned with the permanent magnet flux. Furthermore, we treat the rotor electrical velocity, $\omega_{re}(t)$, as a known (i.e., measured) time-varying exogenous signal. Finally, we note that, under torque regulation, the PMSM plant (1)-(2) constitutes an over-actuated system with two inputs, v_d^r and v_q^r, and a single output, τ, to be regulated, for which the full state, i_d^r and i_q^r, is available for measurement.

B. Statement of the Control Objectives

As it concerns the work presented in this paper, the term "Simultaneous Identification and Control" or "SIC" refers specifically to control methodologies which ensure that inputs to the plant under control are persistently exciting while also simultaneously achieving control objectives such as output regulation. This is to distinguish the work presented herein from standard adaptive control designs which estimate parameters online, but do not guarantee or require parameter convergence, as well as techniques which simply inject a perturbation to provide excitation without special effort to minimize the perturbation's impact on the regulated output.

Accurate parameter knowledge is sometimes desirable for secondary objectives such as condition monitoring, or simply to guarantee that transient specifications are maintained. While there are many advantages to ensuring that inputs are persistently exciting, identification and control are conflicting objectives in the sense that the persistently exciting signals push the system away from the equilibrium point which the control is trying to maintain. However, in the case of over-actuated systems, the input which generates a particular desired output is not unique. This provides an opportunity to simultaneously achieve both identification and control objectives with minimal (if any) compromise, by restricting the excitation to the "null space".

In this paper, we will explore a receding horizon control allocation (RHCA) approach to the simultaneous identification and control (SIC) of an over-actuated plant (i.e., PMSMs) with the specific control objectives:

1) Identification of electrical machine parameters;
2) Accurate PMSM torque regulation.

III. Proposed Control Architecture

The proposed RHCA-SIC design utilizes a two-level structure with reference signals generated by the RHCA being fed to the inner-loop adaptive current regulator, as depicted in Figure 1. The adaptive current regulator ensures fast, accurate tracking of the filtered[2] reference current trajectories, while the "outer-loop" RHCA exploits the over-actuated nature of the PMSM to generate reference currents which are both persistently exciting and produce the desired torque.

A. Inner-loop Controller

The inner-loop controller is a Lyapunov-based adaptive current regulator [20] which has been extended to include PMSMs (i.e., magnetic saliency is considered). The adaptive

[2]By the reference models, $M(s) = \frac{\lambda}{s+\lambda}$.

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

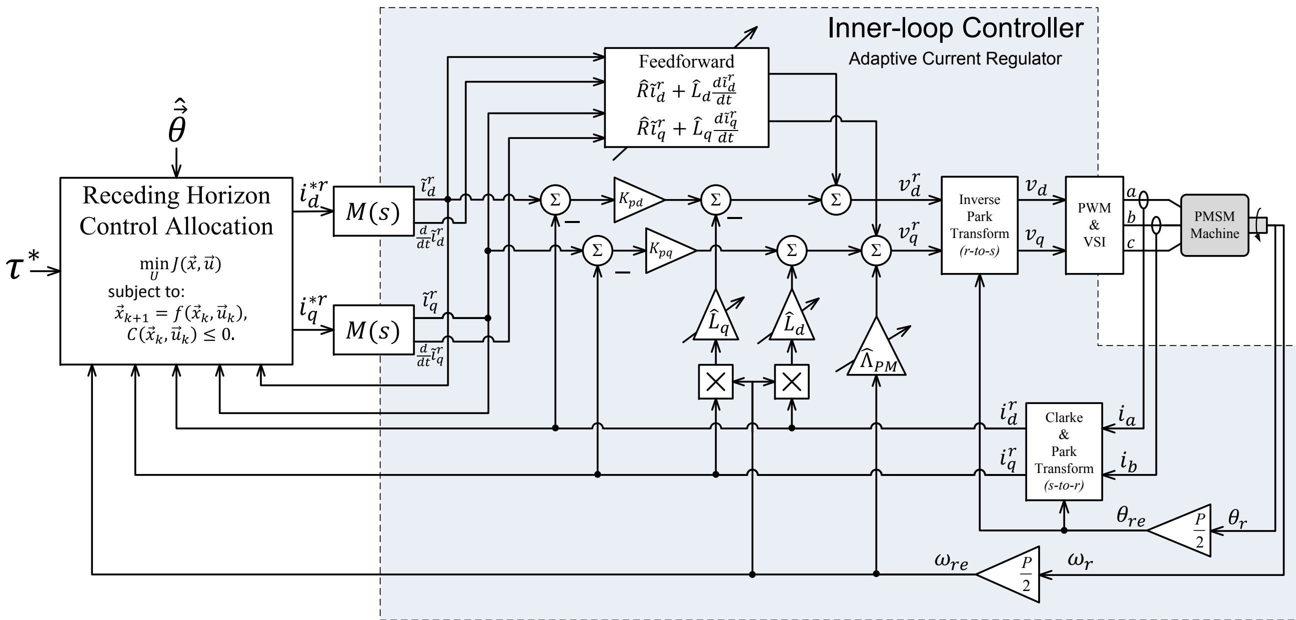

Fig. 1. Block diagram of the proposed RHCA-SIC methodology for PMSM torque regulation.

current regulator ensures that the 2-phase equivalent stator currents asymptotically converge to track the trajectories produced by the reference models. We define the direct and quadrature stator current errors as follows:

$$\begin{aligned} e_{id}^r &= \tilde{i}_d^r - i_d^r, \\ e_{iq}^r &= \tilde{i}_q^r - i_q^r, \end{aligned} \tag{3}$$

where the "tilde" ($\tilde{\cdot}$) denotes filtered reference signals, i.e., the output of $M(s)$.

The control law for our adaptive current regulator uses a mix of feedforward, feedback decoupling, and proportional feedback terms, and is given by

$$\begin{aligned} v_d^r &= \hat{R}\tilde{i}_d^r + \hat{L}_d \frac{d\tilde{i}_d^r}{dt} - \omega_{re}\hat{L}_q i_q^r + K_{pd} e_{id}^r, \\ v_q^r &= \hat{R}\tilde{i}_q^r + \hat{L}_q \frac{d\tilde{i}_q^r}{dt} + \omega_{re}\hat{L}_d i_d^r + K_{pq} e_{iq}^r + \omega_{re}\hat{\Lambda}_{PM}, \end{aligned} \tag{4}$$

where the "hat" ($\hat{\cdot}$) denotes estimated parameters, K_{pd} and K_{pq} are the respective direct and quadrature-axis proportional gains, and the derivative terms are produced by the reference model (i.e., $\tilde{\vec{i}}^r = \{M(s)\}\,\vec{i}^{r*}$ and $\frac{d}{dt}\tilde{\vec{i}}^r = \{sM(s)\}\,\vec{i}^{r*}$, where $M(s)$ is a a stable, minimum phase, proper, unity dc gain, first-order transfer function[3]).

The estimated parameters in (4) are updated via the following adaptive parameter update law

$$\dot{\hat{\vec{\theta}}} = \mathbf{\Gamma}\mathbf{\Phi}\vec{e}_i^{\,r}, \tag{5}$$

where $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0$ is the adaptation gain matrix, $\vec{e}_i^{\,r} = \left[e_{id}^r \;\; e_{iq}^r\right]^T$ is the stator current error vector, and the regressor matrix, $\mathbf{\Phi}$, is given by

$$\mathbf{\Phi} = \begin{bmatrix} \tilde{i}_d^r & \tilde{i}_q^r \\ \frac{d}{dt}\tilde{i}_d^r & \omega_{re} i_d^r \\ -\omega_{re} i_q^r & \frac{d}{dt}\tilde{i}_q^r \\ 0 & \omega_{re} \end{bmatrix}. \tag{6}$$

It can be shown, using Barbalat's lemma [23] and the following Lyapunov function

$$V(\vec{e}_i^{\,r}, \vec{e}_\theta) = \frac{1}{2}\left(\vec{e}_i^{\,rT}\mathbf{L}\vec{e}_i^{\,r} + \vec{e}_\theta^{\,T}\mathbf{\Gamma}^{-1}\vec{e}_\theta\right), \tag{7}$$

that the control law (4) with adaptive update (5) renders the PMSM dynamics (1) stable in the sense of Lyapunov with $\vec{e}_i^{\,r} \to 0$ as $t \to \infty$, where $\mathbf{L} = \mathrm{diag}\left[L_d, L_q\right]$ is a diagonal matrix of the direct and quadrature axis self-inductances, and $\vec{e}_\theta^{\,T} = \left[R \;\; L_d \;\; L_q \;\; \Lambda_{PM}\right] - \left[\hat{R} \;\; \hat{L}_d \;\; \hat{L}_q \;\; \hat{\Lambda}_{PM}\right]$ is the parameter error vector. Convergence of the parameter error follows when the regressor matrix (6) is persistently exciting. Lastly, we note that a "switching σ-modification" [24] is used on (5) for robustness.

B. Control Allocation

The primary objective in any control allocation problem is to find the "best" distribution of control effort among multiple actuators to achieve a desired effect (e.g., generate a "virtual" control input which achieves the desired output). Additionally, by solving the problem online, the effects of actuator saturation and failures can be taken into account. Control allocation is particularly well suited to over-actuated problems which permit the inclusion of secondary objectives, such as control effort minimization.

Typically, the control allocation problem is treated as a static optimization problem, assuming that the "actuator" response is instantaneous [25], [26]. As it concerns

[3] $\{\cdot\}$ denotes a dynamic operator with transfer function "$\cdot$".

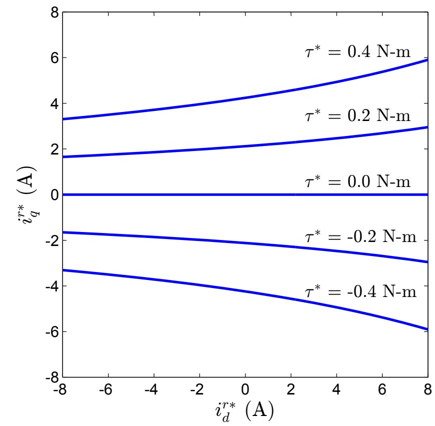

Fig. 2. Sets of current pairs, (i_d^{r*}, i_q^{r*}), which yield various torques for a machine with large saliency (to magnify nonlinearity).

torque control for the (over-actuated) PMSM, the control allocation problem consists of finding a reference current pair, (i_d^{r*}, i_q^{r*}), which produce a desired torque, τ^*. The inner-loop controller, discussed in the previous subsection, is then tasked with producing the voltage pair, (v_d^r, v_q^r), which generates these reference currents. Since the problem is over-actuated, there exist an infinite number of reference currents which yield a given torque. The reference current solution set for some τ^* is described by all pairs $(i_d^{r*}, i_q^{r*}) \in \mathcal{M} := \{(i_d^{r*}, i_q^{r*}) : |\tau^* - h(\vec{i}^{r*}, \vec{\theta})| = 0\}$, and are depicted in Figure 2. In discrete-time, the static control allocation problem for a torque regulated PMSM can be stated as

$$\begin{aligned} \min_{\vec{i}_k^{r*}} \quad & \vec{i}_k^{r*\,T} \mathbf{R} \vec{i}_k^{r*} \\ \text{s.t.} \quad & |\vec{i}_k^{r*}| \leq I_{max}, \\ & |\tau_k^* - h(\vec{i}_k^{r*}, \hat{\vec{\theta}}_k)| = 0, \end{aligned} \tag{8}$$

where our secondary objective is the standard weighted quadratic function of the reference input with $\mathbf{R} > 0$, which minimizes the control effort, and therefore, the ohmic losses as well. While this problem formulation is sufficient for torque regulation, it doesn't ensure persistently exciting reference currents without varying the commanded torque. In the next section, we discuss metrics for persistent excitation and their inclusion in the control allocation problem.

IV. Receding Horizon Control Allocation for Simultaneous Identification and Control

To ensure that the reference currents generated by the control allocation are persistently exciting, we seek a metric which will provide a measure of how persistently exciting the regressor matrix (6) is over some time interval. Such a metric will then be included in the objective (or cost) function of the control allocation problem to encourage the generation of reference signals which are persistently exciting.

A. The Fisher Information Matrix and Persistent Excitation

The identification of parametric models is of interest to a wide variety of disciplines, well beyond that of the control community. In statistics, as well as other fields, the conditioning of the *Fisher information matrix* is used to judge how *informative* an experiment (i.e., its data) is with respect to the identification of a given parametric model. Mathematically, given N discrete observations (i.e., measurements) of a single output[4], $y(t_k)$, at time t_k with $k \in [1 \cdots N]$, of some process described by

$$y(t_k) = \mathcal{H}(t, \vec{\theta}), \tag{9}$$

the Fisher information matrix is defined as

$$\mathbf{F} = \sum_{k=1}^{N} \left(\frac{\partial y(t_k)}{\partial \vec{\theta}} \right)^T \left(\frac{\partial y(t_k)}{\partial \vec{\theta}} \right), \tag{10}$$

where $\vec{\theta}$ is the parameter vector we are interested in identifying. Note that $\mathbf{F}$ is a symmetric positive-semidefinite matrix. When Gaussian noise is considered in the estimation problem formulation, $\mathbf{F}^{-1}$ gives the *Cramer-Rao lower bound* on the achievable covariance of an unbiased estimator. Clearly, if the experiments are not informative, the Fisher information matrix (10) will be poorly conditioned leading to high uncertainty in the parameter estimates.

To provide a measure of the conditioning of the Fisher information matrix, generally for the purpose of "optimal experiment design", the determinant of $\mathbf{F}$ is often used (sometimes referred to as "D-optimality") [27], i.e.,

$$J_D = \det(\mathbf{F}). \tag{11}$$

However, J_D is not a concave function, and so using it for optimal experiment design could lead to sub-optimal solutions given by local maxima. A better choice for the objective function is given by

$$J_D = \log(\det(\mathbf{F})), \tag{12}$$

which has an equivalent maximum to (11), but is non-negative concave [27], [28].

For processes described by a linear parameterization, e.g.,

$$y(t_k) = \vec{\phi}^T(t)\vec{\theta}, \tag{13}$$

the Fisher information matrix (10) simplifies to the familiar form used when defining persistent excitation

$$\mathbf{F} = \sum_{k=1}^{N} \vec{\phi}(t_k)\vec{\phi}^T(t_k), \tag{14}$$

where $\vec{\phi}(t_k)$ is the *regressor* vector. In discrete-time, a bounded vector signal $\vec{\phi}(t_k)$ is said to be *persistently exciting* (PE) if there exists $N > 0$ and $\alpha_0 > 0$ such that

$$\mathbf{F} = \sum_{k=1}^{N} \vec{\phi}(t_k)\vec{\phi}^T(t_k) \geq \alpha_0 \mathbf{I} \tag{15}$$

for all $t_k \geq t_0$ [29].

[4]The extension to multiple outputs is trivial.

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

B. Receding Horizon Control Allocation for Simultaneous Identification and Control

Since the Fisher information matrix (10) becomes singular when evaluated at any given time instant, the addition of a metric for persistent excitation requires modifying the control allocation problem (8) to consider a finite time horizon in the optimization, making a receding horizon (or MPC) framework a natural choice. In the past, researchers have proposed receding horizon control allocation (or MPCA) approaches to account for actuator dynamics, e.g., [30], [31]. In this work, we will utilize the receding horizon control allocation (RHCA) framework to accommodate the addition of the persistent excitation metric (12).

Implementation of a RHCA requires a dynamic model of the inner-loop system to predict future state trajectories and evaluate the regressor matrix for optimization. The prediction model is formulated using the certainty equivalence principle, that is, assuming that the estimated parameters are equal to their true values. This turns out to be a minor assumption, however, because we are assured, that as long as the system is persistently excited, the estimated parameters will converge to their true values. Augmenting the reference filter states, $\tilde{\vec{i}}^r$, with the PMSM states, $\vec{i}^r$, the inner-loop dynamics for prediction are therefore given by

$$\begin{aligned} \dot{\vec{x}} &= \bar{\mathbf{A}}(\hat{\vec{\theta}})\vec{x} + \bar{\mathbf{B}}\vec{i}^{r*}, \\ \vec{z} &= \bar{\mathbf{C}}\vec{x} + \bar{\mathbf{D}}\vec{i}^{r*}, \end{aligned} \tag{16}$$

with,

$$\bar{\mathbf{A}}(\hat{\vec{\theta}}) = \begin{bmatrix} -\left(\hat{R}\mathbf{I} + \mathbf{K}_p\right)\hat{\mathbf{L}}^{-1} & \left(\hat{R}\mathbf{I} + \mathbf{K}_p\right)\hat{\mathbf{L}}^{-1} - \lambda\mathbf{I} \\ \mathbf{0} & -\lambda\mathbf{I} \end{bmatrix},$$

$$\bar{\mathbf{B}} = \lambda\begin{bmatrix}\mathbf{I}\\ \mathbf{I}\end{bmatrix},\ \bar{\mathbf{C}} = -\lambda\left[\mathbf{0}\ \mathbf{I}\right],\ \bar{\mathbf{D}} = \lambda\mathbf{I},$$

where $\vec{x} = [\vec{i}^{rT}\ \tilde{\vec{i}}^{rT}]^T$ is the augmented state vector, $\vec{z} = \frac{d}{dt}\tilde{\vec{i}}^r$ is needed to evaluate the regressor matrix, and so we treat it as an output of the prediction model, and $\lambda > 0$ sets the bandwidth of the (first-order) reference model filters. For the discrete-time implementation, the prediction model (16) is discretized using a zero-order hold.

Since the reference currents, $\vec{i}^{r*}$, have no effect on the estimation of the permanent magnet flux linkage[5], Λ_{PM}, we do not include the bottom row of the regressor matrix (6), which corresponds to the Λ_{PM} term, in our optimization. We define the truncated regressor matrix as

$$\bar{\mathbf{\Phi}}(\vec{x},\vec{z}) = \begin{bmatrix} \tilde{i}_d^r & \tilde{i}_q^r \\ \frac{d}{dt}\tilde{i}_d^r & \omega_{re} i_d^r \\ -\omega_{re} i_q^r & \frac{d}{dt}\tilde{i}_q^r \end{bmatrix}. \tag{17}$$

Assuming the estimated parameters, $\hat{\vec{\theta}}_k$, torque reference, τ_k^*, and rotor electrical velocity, $\omega_{re,k}$, to all be constant over the prediction horizon, the extension of (8) to include a metric (12) for persistent excitation is given by

$$\begin{aligned} \min_{\vec{i}_j^{r*}} \quad & \sum_{j=k}^{k+N-1} \vec{i}_j^{r*\,T}\mathbf{R}\vec{i}_j^{r*} - \rho \log\det\left(\mathbf{F}(\vec{x},\vec{z})\right) \\ \text{s.t.} \quad & \vec{x}_{j+1} = \bar{\mathbf{A}}(\hat{\vec{\theta}}_k)\vec{x}_j + \bar{\mathbf{B}}\vec{i}_j^{r*}, \\ & \vec{z}_j = \bar{\mathbf{C}}\vec{x}_j + \bar{\mathbf{D}}\vec{i}_j^{r*}, \\ & \mathbf{F}(\vec{x},\vec{z}) = \sum_{j=k}^{k+N-1} \bar{\mathbf{\Phi}}(\vec{x}_j,\vec{z}_j)\bar{\mathbf{\Phi}}^T(\vec{x}_j,\vec{z}_j), \\ & |\vec{i}_j^{r*}| \le I_{max},\ \forall\, j \in [k \cdots k+N-1], \\ & |\tau_k^* - h(\vec{i}_j^{r*},\hat{\vec{\theta}}_k)| \le \epsilon,\ \forall\, j \in [k \cdots k+N-1], \end{aligned} \tag{18}$$

where $\mathbf{R} \ge 0$ is the input weighting matrix, $\rho \ge 0$ is the PE metric weighting, and $\epsilon > 0$ determines the maximum allowable perturbation in the regulated torque output. While the constraint on the regulated output error could be included in the objective function and penalized[6], the over-actuated nature of our problem permits the use of it as a constraint[7]. We do, however, include it here as a "relaxed" (i.e., inequality) constraint to speed up the numerical optimization, help ensure that a feasible solution exists, and allow for small perturbations in regulated output if it will aid the parameter identification.

C. The Crucial Role of Past Input and State Data

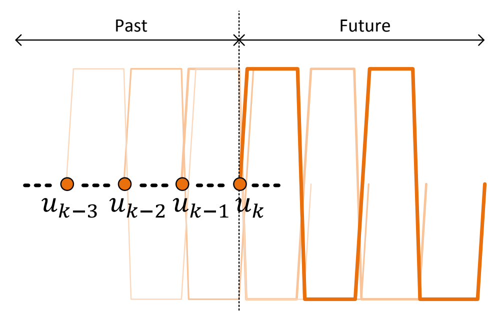

Fig. 3. Disregard for past input (and state) data leading to a lack of persistent excitation.

To highlight the importance of incorporating recent past input and state data in the calculation of the Fisher information matrix for maximizing excitation in the receding horizon framework, imagine conditions are such that the optimal predicted input trajectory is the same at every subsequent time step. In the receding horizon (or MPC) framework, only the first step of the optimal sequence is applied at any given time step. So while the optimal predicted sequence may be persistently exciting, the actual sequence applied to the system is very much not persistently exciting. This is depicted

[5]It can be shown that identification of Λ_{PM} only requires a non-zero rotor velocity, i.e., $\omega_{re} \ne 0$.

[6]This approach was briefly investigated in numerical simulations, but was found to require very large penalties to achieve reasonable tracking performance which could lead to numerical conditioning issues.

[7]Since we know that, under normal operating conditions, solutions satisfying $|\tau_k^* - h(\vec{i}_{k+i}^{r*},\hat{\vec{\theta}}_k)| \le \epsilon$ exist.

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

graphically in Figure 3. When past data is considered, it is clear that the first time step in each subsequent optimal sequence, which will be applied to the system, must differ from the previous to ensure that persistently exciting inputs are indeed generated.

With this issue in mind, we modify the RHCA problem proposed in (18) to include N_p points of recent (past) data (i.e., the last N_p values of the states and inputs) in addition to the usual prediction horizon, N_f:

$$\begin{aligned} \min_{\vec{i}_j^{r*}} \quad & \sum_{j=k}^{k+N_f-1} \vec{i}_j^{r*\,T} \mathbf{R} \vec{i}_j^{r*} - \rho \log\det\left(\mathbf{F}(\vec{x}, \vec{z})\right) \\ \text{s.t.} \quad & \vec{x}_{j+1} = \bar{\mathbf{A}}(\hat{\vec{\theta}}_k)\vec{x}_j + \bar{\mathbf{B}}\vec{i}_j^{r*}, \\ & \vec{z}_j = \bar{\mathbf{C}}\vec{x}_j + \bar{\mathbf{D}}\vec{i}_j^{r*}, \\ & \mathbf{F}(\vec{x}, \vec{z}) = \sum_{j=k-N_p}^{k+N_f-1} \bar{\mathbf{\Phi}}(\vec{x}_j, \vec{z}_j)\bar{\mathbf{\Phi}}^T(\vec{x}_j, \vec{z}_j), \\ & |\vec{i}_j^{r*}| \le I_{max}, \forall\, j \in [k \cdots k + N_f - 1], \\ & |\tau_k^* - h(\vec{i}_j^{r*}, \hat{\vec{\theta}}_k)| \le \epsilon, \forall\, j \in [k \cdots k + N_f - 1]. \end{aligned} \tag{19}$$

The change is subtle, but the effects are profound, as will be demonstrated in the simulation results to follow.

Finally, to reduce the dimension of the numerical optimization problem, a linear B-spline is used to approximate the control input [7]. For the purpose of trajectory optimization, the reference currents on the time interval $t_k \in [0, T]$ are given by

$$i_{d,q}^{r*}(t_k) = \sum_{j=0}^{J} \alpha_j \mathcal{B}\left(\bar{t}_k\right), \tag{20}$$

where $\bar{t}_k$ is the normalized time sequence, given by

$$\bar{t}_k = \frac{t_k}{T}\frac{J-1}{2} - \frac{j-1}{2},$$

and $\mathcal{B}(\bar{t}_k)$ are the triangular basis functions,

$$\mathcal{B}(\bar{t}_k) = \begin{cases} 1 - 2|\bar{t}_k| & \text{for } |\bar{t}_k| \le 0.5, \\ 0 & \text{otherwise,} \end{cases}$$

which are precomputed and stored in memory. Thus, we optimize over a vector of the weighting coefficients, α_j, rather than the full resolution time sequence.

Note that a sufficient number of "knot" points (i.e., sufficiently large J) must be used with respect to the length of the time interval, T, to ensure that signals are approximated with sufficient fidelity. The advantages of using a linear spline are that constraints can be enforced simply by looking at the weighting coefficients, α_j, which give the signal value at the knot points[8] and a reduction in the dimension of the optimization problem, speeding up the numerical optimization.

[8]Higher-order polynomial basis functions can lead to "peaking" and constraint violation.

V. Simulation Results

Numerical simulations using Matlab/Simulink are used to verify the effectiveness of the proposed receding horizon control allocation methodology for SIC of PMSMs. The simulations capture the sampled-data nature of a practical implementation by implementing the controller in a triggered subsystem which runs at 8 kHz for the inner-loop (high-bandwidth) adaptive current regulator and a quarter of that (i.e., 2 kHz) for the RHCA, while the machine dynamics are solved using *ode45*. An ideal "average-value" inverter model is assumed, that is, the voltage commands generated by the controller are fed directly into the PMSM model. The optimization problem is solved using the active-set algorithm in *fmincon*, and the simulation parameters in Table II were used in all simulations except where otherwise noted.

TABLE II
Simulation parameters.

Description	Value
Electrical Machine Parameters:	
R	109 $m\Omega$
L_d	192 μH
L_q	212 μH
Λ_{PM}	12.579 mV-s
P	10
Control Design Parameters:	
K_{pd}, K_{pq}	0.2
$\mathbf{\Gamma}$	diag([30 30 30 30])
λ	225
$\mathbf{R}$	$0.1 \cdot \mathbf{I}$
ρ	10
Prediction Horizon, N_f	25
No. of Recent Data Points, N_p	25
Simulation Settings:	
Solver	ode45
Max Step Size	25 μ-sec

A. Static Control Allocation

For completeness, simulation results for the static control allocation problem (8) are provided in Figure 4. Inspection of the results in Fig. 4 reveals that, without a metric for excitation, the control allocation algorithm is simply trying to track the desired torque command using a minimal amount of control effort. Thus, the commanded direct-axis current is essentially zero for the entirety of the simulation, corresponding to a minimal current magnitude operating point. The lack of excitation leads to slow parameter convergence, since excitation is only provided by the step changes in torque. Additionally, the lack of accurate parameter knowledge leads to a small but undesirable overshoot in the transient torque responses (see "zoomed" plots in Fig. 4). Finally, the steady-state tracking is expected given that the magnetic parameters more or less converge to their true values, and the inner-loop

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

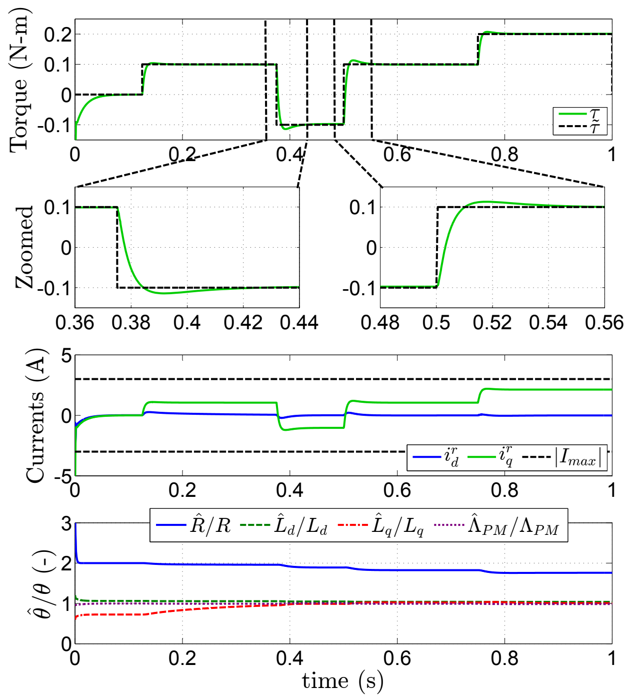

Fig. 4. Simulation of the static control allocation (8) without PE maximization.

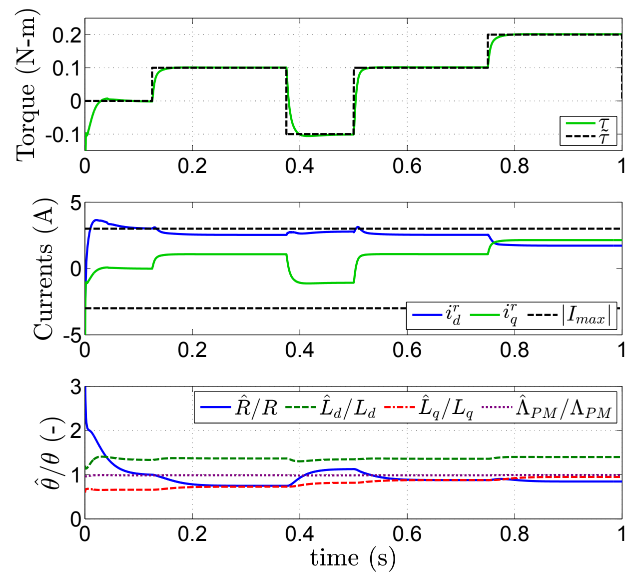

Fig. 5. Simulation of the RHCA with PE maximization and *without* past input and state data (18).

controller is designed to guarantee asymptotic convergence of the stator current error regardless of the accuracy of the parameter estimates.

B. RHCA-SIC without Past Input and State Data

For completeness, simulation results for the RHCA-SIC problem formulation when past input and state data is disregarded (18) are provided in Figure 5. Inspection of the results in Fig. 5 reveals that, while the controller does do a good job of tracking the desired torque, the RHCA algorithm fails to generate persistently exciting signals. The lack of persistently exciting inputs once again leads to parameter stagnation.

C. RHCA-SIC with Past Input and State Data

When past input and state data are included in the RHCA (19), we see that all of the parameters converge to their true values, as the simulation results in Figure 6 demonstrate. Inspection of the results in Fig. 6 reveals that, not only does the PE metric with past data generate persistently exciting reference currents, but the overall RHCA-SIC strategy takes advantage of the over-actuated nature of the plant by utilizing the direct-axis current, which has a small impact on the torque production, for the majority of the excitation. Meanwhile, the quadrature-axis current is primarily used to satisfy the torque regulation (i.e., control) objective, agreeing with our intuition about the SIC problem for PMSMs [20], [21]. Note that while the torque output is initially perturbed by the additional excitation, this perturbation vanishes asymptotically as the parameter estimates converge to their true values. This happens because accurate parameter knowledge is needed in order to accurately define the set $\mathcal{M}$ in which the states may vary. Additionally, the losses incurred due to the excitation may be reduced by increasing the penalty, $\mathbf{R}$, accordingly. Lastly, we note that a distinct advantage of the proposed optimization-based RHCA-SIC methodology is that it can easily handle other plants where the allocation strategy for SIC is not so intuitive.

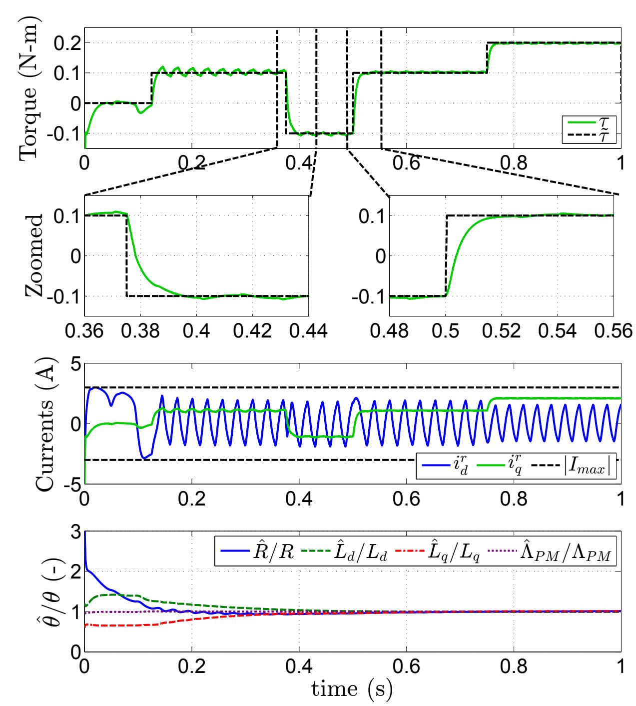

Fig. 6. Simulation of the proposed RHCA-SIC methodology for over-actuated systems with PE maximization and past data (19).

Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.

VI. Conclusion

In this paper, we have presented an optimization-based simultaneous identification and control methodology for PMSMs which exploits the over-actuated nature of the machine. A receding horizon control allocation (RHCA) framework is used which includes a metric for maximizing the excitation characteristics of the generated reference current trajectories. The RHCA feeds the computed reference currents to a lower-level adaptive current regulator which ensures asymptotic tracking of a reference model. The importance of including past input and state data in the RHCA-SIC algorithm is discussed, and simulation results demonstrating the effectiveness of the proposed RHCA-SIC methodology with past input and state data are presented, as well as scenarios without PE maximization and which disregard past data.

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Preprint submitted to 2016 American Control Conference.
Received September 28, 2015.